Section 8.3
Relative Rates of Growth

Let f(x) and g(x) be positive for x sufficiently large.

1.) f grows faster than g (and g grows slower than f) as $x \to \infty$ if

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty \text{, or equivalently, if}$$

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$$

2. f and g grow at the same rate as $x \to \infty$ if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L \neq 0.$$

L is finite and not zero!

Example.....

Which grows faster

y = 2x or y = x?

$$\lim_{x \to \infty} \frac{2x}{x} = \lim_{x \to \infty} 2 = 2$$

Show that the function e^x grows faster than x^2 as $x \to \infty$

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$$

① $$\lim_{x \to \infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$$

② $$\lim_{x \to \infty} \frac{e^x}{2} = \infty$$

Show that ln x grows slower than a.) x and b.) x^2 as $x \to \infty$

@ $\lim_{x \to \infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$

① $\lim_{x \to \infty} \frac{1}{x} = 0$

ⓑ $\lim_{x \to \infty} \frac{x^2}{\ln x} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{2x}{\frac{1}{x}} = 2x^2 = \infty$

Show that x grows at the same rate as x + sin x as x →∞

$$\lim_{x\to\infty} \frac{x+\sin x}{x} \qquad \frac{1}{1+\cos x}$$

$$\lim_{x\to\infty} \frac{x}{x} + \frac{\sin x}{x}$$

$$\lim_{x\to\infty} 1 + \frac{\sin x}{x} = 1 + 0 = 1$$

Let a and b be numbers greater than 1. Show that $\log_a x$ and $\log_b x$ grow at the same rate as $x \to \infty$.

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \lim_{x\to\infty} \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln b}}$$

$$= \lim_{x\to\infty} \frac{\ln b}{\ln a}$$

$$= \frac{\ln b}{\ln a}$$

Transitivity of Growing Rates

If f grows at the same rate as g as x $\to\infty$

and g grows at the same rate as h as x $\to\infty$,

then f grows at the same rate as h as x $\to\infty$.

Show that $f(x) = \sqrt{x^2 + 5}$ and $g(x) = (2\sqrt{x} - 1)^2$

grow at the same rate as $x \to \infty$.